Train simulation and headway calculations: an approach based on parametrised continuous curves

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Abstract

The project referred to in this paper consists of three phases. First, the development of a very accurate simulator in order to analyse the train and track performance of a balanced line, to be used as a CAD tool. Second, the development of a very accurate simulator in order to analyse the behaviour of a line under train-to-train disturbances and different passenger-flow scenarios. Third, the development of a control system to guarantee a constant time interval between trains, using the accurate simulator of the second phase as a workbench for case studies.

This paper describes in detail the first phase of the project. As a result of this work, a software tool (PACA) is currently being used at DIMETRONIC S.A. as a CAD tool for the analysis and design of control and protection systems. PACA is implemented on a PC environment and written in ANSI-C language.

1 Introduction

The work described in this paper is the result of a long-term co-operation between the R+D department of DIMETRONIC S.A. and a research institute (IIT). This co-operation has lead to a three-year project on simulation and control of railway systems, including both the search for innovative theoretical concepts and specific-application software development. This paper describes in detail the initial phase of the project.

The requirements for this phase of the project covered two different aspects. First, the development of an accurate and user-friendly CAD tool for the analysis and design of protection-control systems, given a railway line under balanced conditions. Second, the reusability of models and concepts for the realistic simulation of the behaviour of the system under unbalanced situations [1].

The consideration of these two aspects leads to the definition of the following requirements:

- Accurate models of the rolling stock, the track characteristics, different protection systems and control strategies.
- Efficient and precise storage/retrieval of the "history" of the train movement along the line, both for its graphical representation and for allowing asynchronous simulations [1].
- Acceptable performance in terms of response time and memory requirements.
- User-friendly graphical environment for data input and results output.
- Databases management, including consistency checking, versions management and a flexible customer-configurable behaviour.

2 Modelling assumptions

Within this initial phase, the study is focused on simple circular lines (no junctions) with optional headway stations in which the movement of the trains is reversed. The line is assumed to be in balanced conditions, meaning:

- Constant voltage.
- Constant percentage of energy recovered by the line with respect to the energy generated by the trains in the braking processes.
- Constant dwell time (different for each station).
- Constant load of passengers in the trains.
- All the trains are identical.
- No interferences between trains, i.e., the protection systems do not affect the movement of the trains.

With respect to the track characteristics that affect the mechanical equations of the train movement, the following assumptions are made:

- The mass of the train is reduced to its centre of masses (errors can be neglected using typical gradients and train lengths).
- The resistance due to track curves is expressed as an equivalent gradient, dependent on the radius of each curve.
- The air resistance is expressed as a parabolic function of the speed related to the type of train. The tunnel effect is added by means of a track-related factor multiplying the three terms of the parabola (equal to one at open air, bigger than one in a tunnel).

Concerning the model of the motors, the speed/maximum-force curves for both the traction and the braking processes are represented as lists of points, using linear interpolation for intermediate values. For energy consumption calculations, only the currents corresponding to the speed/maximum-force curves at nominal voltage are assumed to be known (also as lists of points). Since the motor of a train do not necessarily operate at maximum force, energy

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consumption is estimated through a special process based on efficiency interpolation, explained in detail in section 5.

The train is driven by acceleration commands, which are automatically translated to force commands by the train equipment depending on its current mass. The control commands consist of successive series of traction-coasting cycles plus different braking parabolas when approaching a station.

The different protection systems have been very accurately modelled in order to perform minimum headway calculations. They include track circuits (ATP), fixed-block, signals and moving-block.

3 Basic movement equation

Under the assumptions described in the previous section, the dynamic equation for the movement of a train is:

$$M \frac{dv}{dt} = m\,v + n - F_g - k\,(a\,v^2 + b\,v + c) \qquad (1)$$

where

M	total mass of the train
v	velocity (speed)
$m\,v + n$	motor force (linear function of velocity)
F_g	force due to the equivalent track gradient
k	tunnel effect factor
$a\,v^2 + b\,v + c$	parabolic expression of air resistance in open air

Given a motor force equation, a constant gradient and a constant tunnel factor, the train speed will follow the basic differential equation:

$$\frac{dv}{dt} = A\,v^2 + B\,v + C \qquad (2)$$

The analytical solutions of this equation have been parametrised depending on the values of $\{A,B,C,v_0\}$, being v_0 the initial speed of the train. The different forms of the solution have been classified into ten "types" of equations. Furthermore, for each type of equation six explicit expressions have been found, corresponding to the functions $\{v(t), t(v), s(t), t(s), v(s), s(v)\}$ (being "s" the space).

These expressions make it possible to have fast-response calculations in all the algorithms for any kind of value request. Furthermore, the fact that the functions $\{v(t), v(s)\}$ are monotonous allows the use of efficient algorithms in finding intersections between movement curves.

4 Simulation technique: events plus parametrised equations

As long as the parameters {A,B,C} remain constant for a given train, its continuous state {v,s} can be found at every instant $t > t_0$ as a function of both the parameters and the initial conditions $\{v_0,s_0,t_0\}$. The "discrete state" of the train may be thus defined by the variables $\{v_0,s_0,t_0,A,B,C\}$; these state variables do not change along time unless some event occurs that changes the values of the parameters {A,B,C}.
The state variable "c" (accumulated energy consumption) is added in order to perform, store and retrieve energy calculations. This requires that the discrete state variables be defined as $\{v_0,s_0,c_0,t_0,A,B,C,m,n\}$, see equation (1).
The result of the simulation of the continuous movement of a train can therefore be expressed as a succession of discrete states, each one representing a parametrised equation (see figure 1).

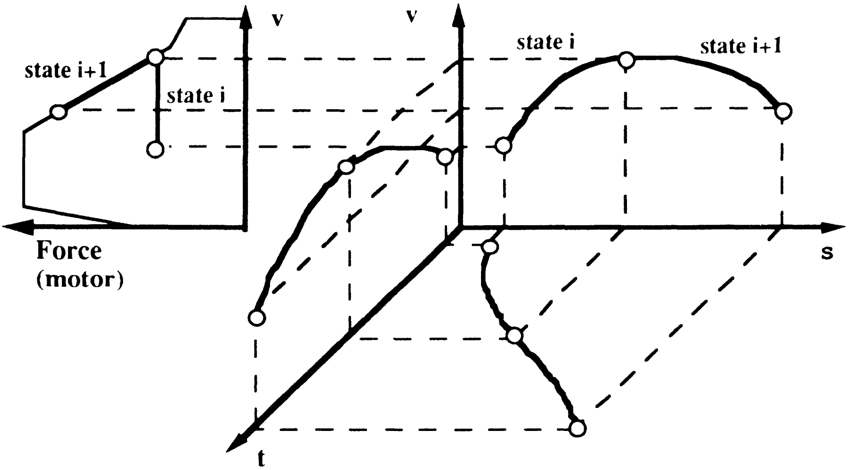

Figure 1. Continuous movement expressed as a list of discrete states.

An event is any instantaneous occurrence that changes the discrete state of the system [2], i.e., that changes the value of the parameters {A,B,C}. The change of the rest of the variables of the new state $\{v_0,s_0,c_0,t_0,m,n\}$ is a function of both the event and the previous state.
The following sources of events have been considered in the model:

- The track generates {s} events (gradients, tunnel effect factors) and {s,v} events (speed limits, including the approaching curves to station stops).
- The motor generates {v} events (change of force/speed expression).
- The control system generates {v}, {s} and {t} events.
- The protection systems generate {v,s,t} events (used only in [1] for unbalanced systems).

The main feature of this simulation technique is an efficient storage and retrieval of simulation results without loss of precision in the continuous state variables {v,s,c}. This allows the performance of accurate headway calculations as well as a variable graphical-display resolution (zooms). The use of explicit expressions for all the functions {v(t), t(v),...} yields very efficient algorithms in terms of computing time.

The ability to store and retrieve the "history" of train movements for a given time interval provides the basis for the asynchronous simulation of a whole system described in [1]. The parametrised equations that compose that history makes it possible to handle time concepts in a simple and flexible way (which is not a trivial question, see [3]).

5 Energy consumption calculations

As it was stated in section 2, only the currents corresponding to the speed/maximum-force curves at nominal voltage are assumed to be known. However, energy consumption has to be estimated somehow for situations in which the motor is not working at the maximum-force curve.

The idea is to find the efficiency of the motor for some relevant working points, based on the available information, and then perform a linear interpolation in the rest of the working points. The question is how to find a variable with respect to which the efficiency of the motor is likely to be linear.

The solution of this problem is shown here as an example for the traction characteristics of the motor. The braking characteristics require a different definition of efficiency.

The maximum force "F", and the corresponding line-current "I_L" for "F" at the nominal voltage "V", are assumed to be known as functions of the speed "v". Assuming the energy losses to be proportional to the square of the motor-current "I" and the force to be proportional to "I", the efficiency "eff" of the motor is:

$$\frac{1}{\mathrm{eff}} = \frac{F\,v + k\,I^2}{F\,v} = \frac{F\,v + k'\,F^2}{F\,v} = 1 + k'\,\frac{F}{v} \qquad (3)$$

Therefore, F/v is the variable with respect to which the 1/eff is linear. On the other hand, different values of 1/eff can be found as a function of this variable, since the following expression depends on the line current "I_L" :

$$\frac{1}{\mathrm{eff}} = \frac{V\,I_L}{F\,v} \qquad (4)$$

The procedure is as follows: given the voltage V, some {F,v} points and some {I_L,v} points at maximum force, several {1/eff,F/v} points are computed using equation (4). For any other working point, 1/eff is found by linear interpolation as a function of F/v, a reasonable approximation according to equation (3).

6 Minimum headway and time margin calculations

Given the history of the movement of a train A between two points of a line, the goal is to find the minimum time delay with which a similar train B may follow train A without detecting its presence. This is the definition of minimum headway applied in this project, valid for any protection system.

The concept of "detection" depends on the particular protection system. There have been modelled four types of such systems: ATP, fixed-block, signals and moving-block:

- In the ATP and fixed-block systems, the presence of a train is detected when, while entering a certain track circuit, a restrictive speed code is received.
- In a signals system, detection means that the nominal movement of the train is affected by a restrictive status of a signal (considering some safety margin).
- In a moving-block system, a train A detects the presence of a train B when the current braking distance of train A (computed with some safety-requirements criteria) reaches the current location of train B.

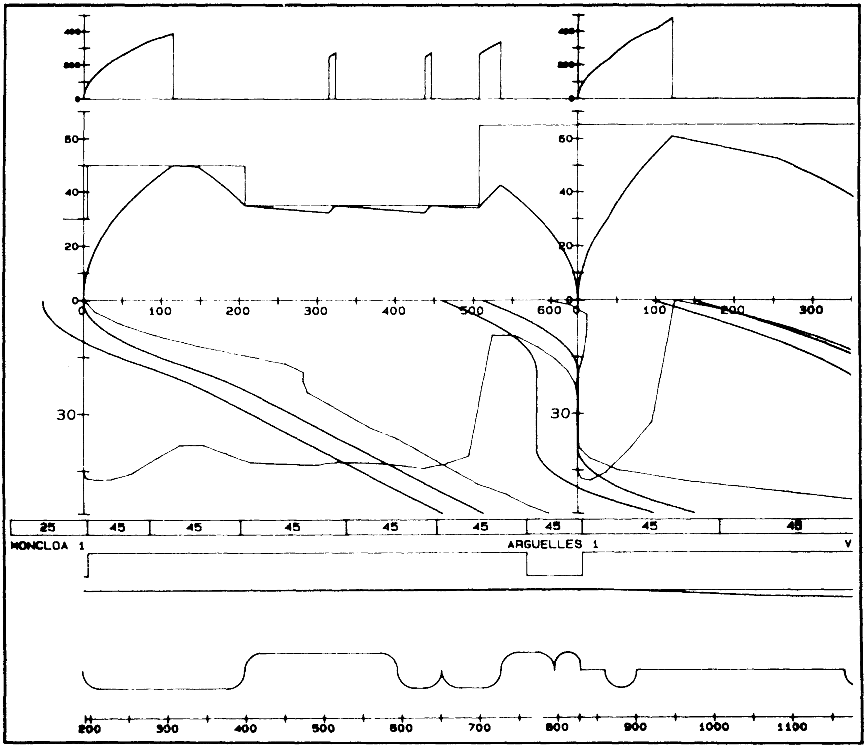

Figure 2. PACA graphical results

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Time margin is a very useful for designing the implementation of the protection systems, in particular for deciding the location and length of track circuits and the signals location. Given a train A that follows another train B, the time margin at a given point of the track is defined as how far (in time) is train A from detecting the presence of train B. Thus, the point of the track with the minimum time margin is the one that defines the minimum headway.
The value of the minimum headway depends on many factors: the protection system implementation, safety margins, track characteristics, speed profiles, braking capabilities, dwell times, and train length. The areas in which the time margin is small require a more careful design of the protections, while those in which the time margin is big require less protection devices.

7 The tool PACA

The tool performs simulations and analysis of the movement of a train along a given line. It generates reports and graphical representation of the simulations, including control commands, headway calculations and energy consumption.
As a CAD tool it provides assistance to the design of energy-saving control commands and the implementation of different protection systems. It is also a user-friendly environment in which data bases can be easily accessed and updated, including a detailed consistency checking.
Figure 2 shows the graphical representation of the results of a simulation. From top to bottom, it includes -as functions of space- line current, speed, time, braking distance, time margin, track circuits, signals, platforms, tunnel effect factor, track profile and track curves.

8 Conclusions

The simulation technique described in this paper has proven to be very accurate and useful for different applications. As a CAD tool, it is flexible and user-friendly, and it is currently being used in several projects and proposals. Further developments include the automatic optimal design of control commands and the automatic optimal location of protection devices.

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